Longford
C of E Primary School

Calculation Policy

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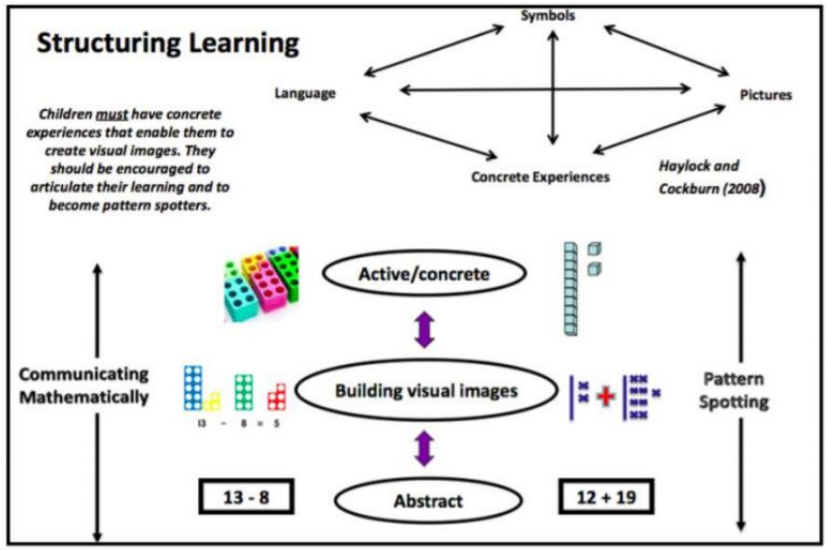
Indicate which legislation or statutory guidance document requires this Policy	

This document outlines for Longford Primary School both the **mental** and **written** methods that should be taught from Year 1 to Year 6. EYFS follow the early years curriculum and a good level of development. Maths assessed as developmental milestones towards a good level of development. It has been devised to meet requirements of the National Curriculum for the teaching and learning of mathematics.

This policy has been designed to teach children through the use of concrete, pictorial and abstract methods. This calculation policy should be used to support children to develop a deep understanding of number and calculation. Background This policy has been developed by Longford Primary School using a range of sources including White Rose calculation guidance and Mathematics Mastery policy as well as other schools with a specific interest in the use of methods to develop number awareness and fluency. The policy only details the strategies; teachers must plan opportunities for pupils to apply these; for example, when solving problems, or where opportunities emerge elsewhere in the curriculum. Using the concrete-pictorial-abstract approach: Children develop an understanding of a mathematical concept through the three steps (or representation) of concrete-pictorial-abstract approach. Reinforcement is achieved by going back and forth between these representations. Concrete representation- The enactive stage - a pupil is first introduced to an idea or a skill by acting it out with real objects. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding. Pictorial representation- The iconic stage - a pupil has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem. Abstract representation- The symbolic stage - a pupil is now capable of representing problems by using mathematical notation, for example: 12 ÷ 2 = 6.



Guidance:

This document provides guidance and examples for key objectives for each year group but is not to be followed as a complete planning aid as not all objectives are exemplified. A child may be working in Year 4 but may not be working towards that year group expectation and therefore will be working on strategies below. For this

reason the policy outlines strategies used for each operation but the strategies have not been headed by the year group children work in. Year group objectives for each operation have been included as guidance as to what children need to know by what year. Methods of calculation are specifically taught and children use them when problem solving.

Aims of the Policy

- To ensure consistency and progression in our approach to calculation
- To ensure that children develop an efficient, reliable, formal written method of calculation for all operations
- To ensure that children can use these methods accurately with confidence and understanding
- To ensure that children become fluent in mathematics and that teachers can adapt lesson to incorporate mastery teaching

How to use the Policy

- Use the policy as the basis of your planning but ensure you make adaptations according to the needs of pupils in your class.
- If, at any time, children are making significant errors, return to the previous stage in calculation
- It is expected that during periods of new learning teachers may extend the teaching sequence in order to embed the key skills
- Always use suitable resources, models and images to support children's understanding of calculation and place value, as appropriate

Addition and Subtraction National Curriculum Objectives for Addition and Subtraction

Year 1:

- read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including zero solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ☐ – 9.

Year 2

- solve problems with addition and subtraction: - using concrete objects and pictorial representations, including those involving numbers, quantities and measures - applying their increasing knowledge of mental and written methods
- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including: - a two-digit number and ones - a two-digit number and tens - two two-digit numbers - adding three one-digit numbers

- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

Year 3

- add and subtract numbers mentally, including: - a three-digit number and ones - a three-digit number and tens - a three-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answer
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Year 4

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

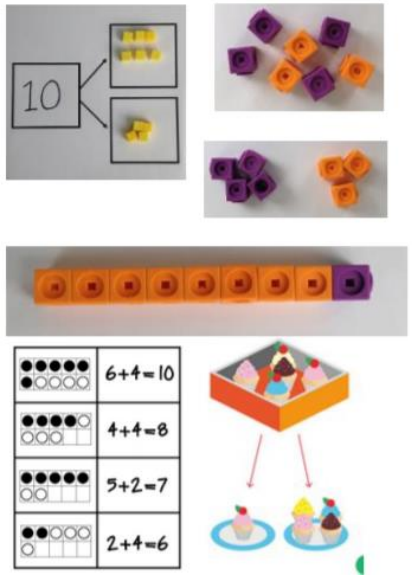
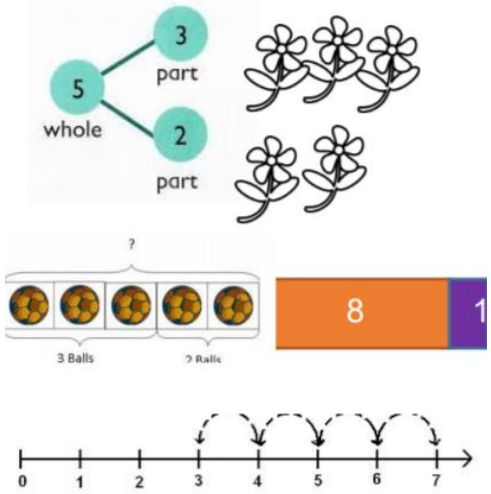
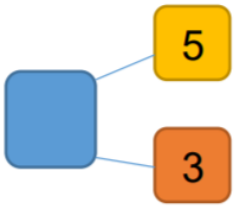
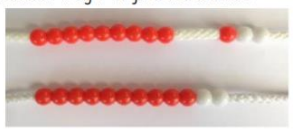
Year 5

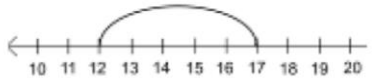
- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

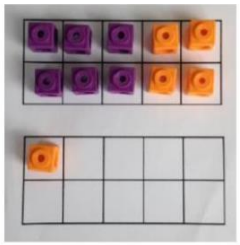
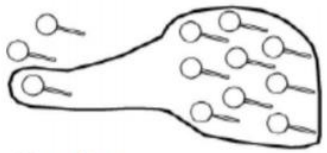
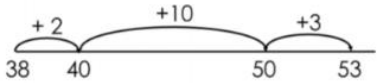
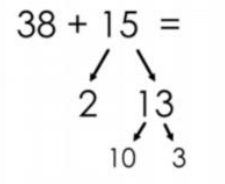
Year 6

- perform mental calculations, including with mixed operations and large numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition and subtraction
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

Progression in skills

Addition			
Objectives and strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole part: part-whole Model Teach both addition and subtraction alongside each other, as pupils will use this model to identify the inverse link between them. This model begins to develop the understanding of the commutativity of addition, as pupils become aware that the parts will make the whole in any order	Use cubes to add two numbers together as a group or in a bar 	Use pictures to add two numbers together as a group or in a bar. 	Use the part-part whole diagram as shown below to move into the abstract. 5 + 3 = 8 10 = 6 + 4 10 - 6 = 4 10 - 4 = 6 10 = 4 + 6 
Starting at the bigger number and counting on As a strategy, this should be limited to adding small quantities only (1, 2 or 3) with pupils understanding that counting on from the	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. 		5 + 12 = 17 17 = 12 + 5 Place the larger number in your head and count on the smaller number to find your answer.

<table>
<tr>
<td>greater is more efficient.</td>
<td></td>
<td>12 + 5 = 17
 

Start at the larger number on the number line and count on in ones or in one jump to find the answer.
  8 + 1 = 9</td>
<td></td>
</tr>
<tr>
<td>Regrouping to make
10.
This is an essential skill that will support column addition later on.
The colours of the beads on the bead string make it clear how many more need to be added to make ten.</td>
<td> 
6 + 5 = 11
Start with the bigger number and use the smaller number to make 10.
Empty spaces on a ten frame make it clear how many more are needed to make 10.</td>
<td>Use pictures or a number line. Regroup or partition the smaller number to make 10.
 
3 + 9 =
 
</td>
<td>7 + 4= 11
If I am at seven, how many more do I need to make 10. How many more do I add on now?
 
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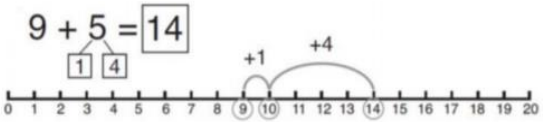
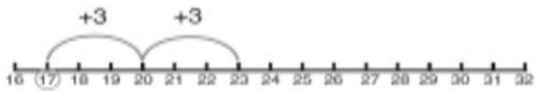
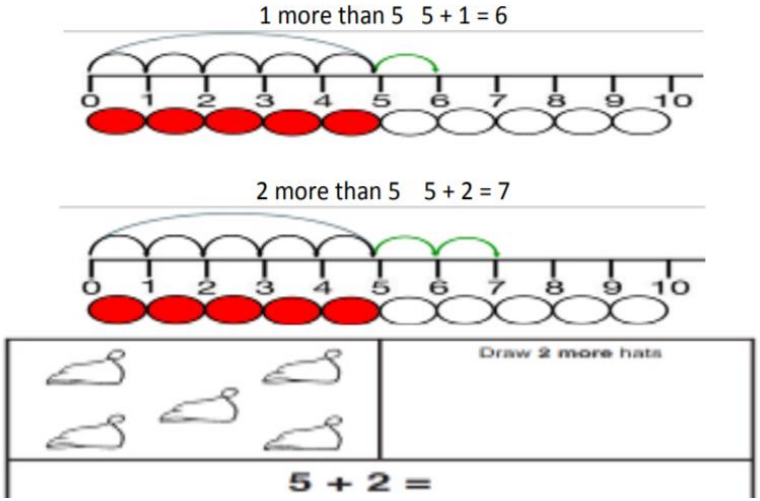
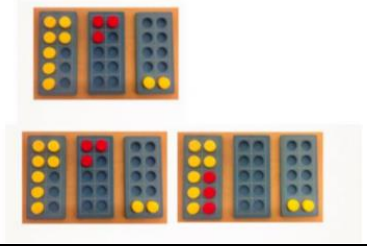
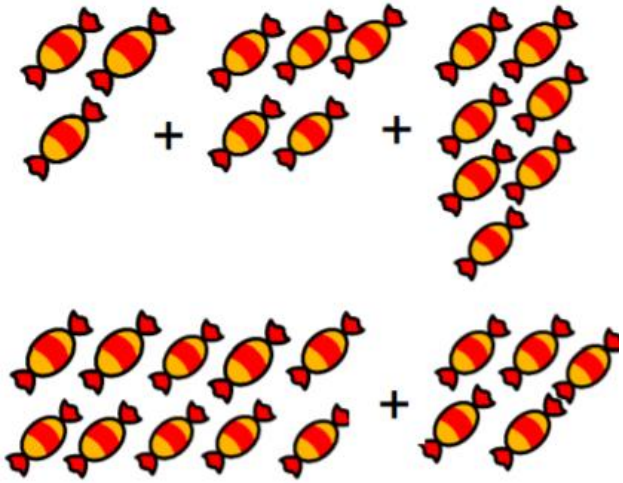
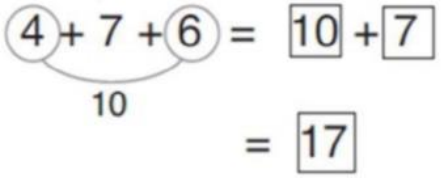
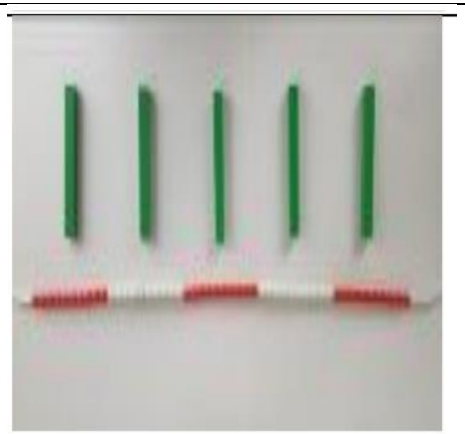
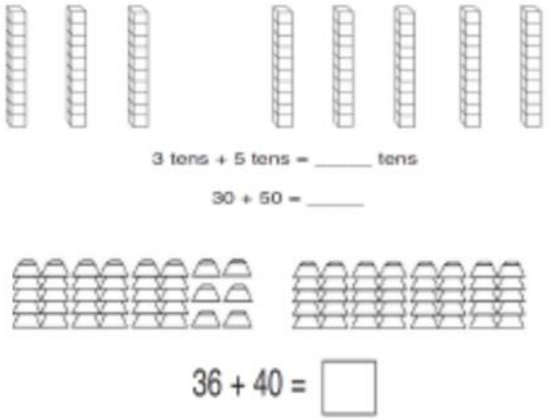
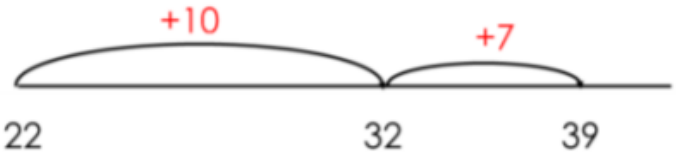
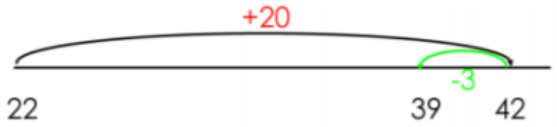
<td> 

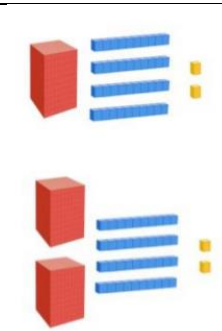
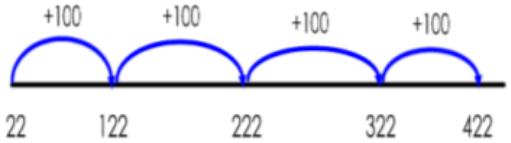
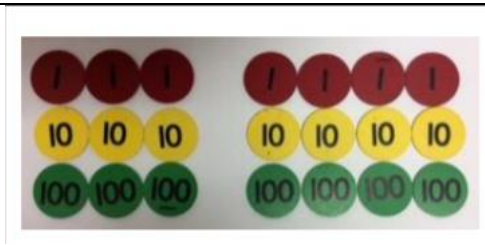
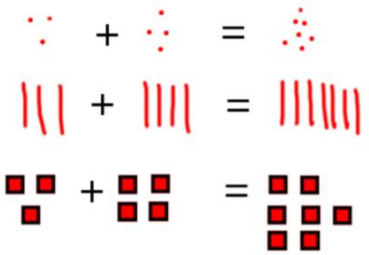
 

17 + 6 = 23
3
3</td>
<td></td>
</tr>
<tr>
<td>Adding 1, 2, 3 more
Here the emphasis</td>
<td></td>
<td></td>
<td></td>
</tr>
</table>

<table>
<tr>
<td>should be on the language rather than the strategy. As pupils are using the bead string, ensure that they are explaining using language such as;
‘1 more than 5 is equal to 6.’
‘2 more than 5 is 7.’
‘8 is 3 more than 5.’</td>
<td>2 more than 5
2 + 5 = 7
5 + 2 = 7
  </td>
<td> 
</td>
</tr>
<tr>
<td>Adding three single digits Pupils may need to try different combinations before they find the two numbers that make 10. The first bead string shows 4, 7 and 6. The colours of the bead string show that it makes more than ten. The second bead string shows 4, 6 and then 7. The final bead string shows how they have now been put together to find the total.</td>
<td>4 + 7 + 6= 17
Put 4 and 6 together to make 10. Add on 7.
 
Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.
7+3+2 = leads to 10 + 2 =
  </td>
<td> 
Add together three groups of objects. Draw a picture to recombine the groups to make 10.</td>
<td> 

Combine the two numbers that make 10 and then add on the remainder.</td>
</tr>
</table>

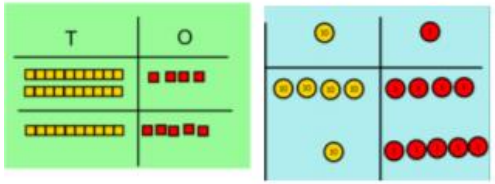
Adding multiples of ten 50 = 30 + 20 Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important, as pupils need to understand that it is a ten and not a one that is being added. It also emphasises the link to known number facts. E.g. '2 + 3 is equal to 5. So 2 tens + 3 tens is equal to 5 tens			
Partitioning one number, then adding tens and ones Pupils can choose themselves which of the numbers they wish to partition. Pupils will begin to see when this method is more efficient than adding tens and taking away the extra ones, as shown.			22 + 17 = 39
Rounding one number, then adding the tens and taking away extra ones Pupils will develop a sense of efficiency with			22 + 17 = 39

this method, beginning to see when rounding and adjusting is more efficient than adding tens and then ones			
Counting on in hundreds and thousands As pupils become familiar with numbers up to 1000, place value should be emphasised and comparisons drawn between adding tens, hundreds and thousands, including use of concrete manipulatives and appropriate images			100 + 39 = 139 39 + 100 = 139 139 = 100 + 39 139 = 39 + 100
Using known facts 3 + 4 = 7 Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy.			

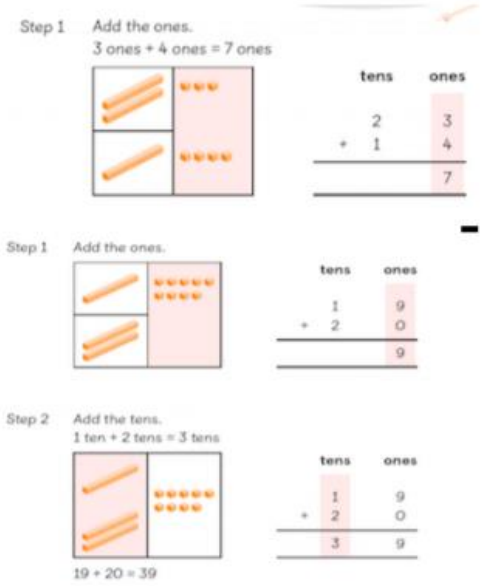
Column method- no regrouping
Place value grids and Dienes blocks should be used as shown in the diagram before moving onto the pictorial representations.
Dienes blocks should always be available, as the main focus in Year 1 is the concept of place value rather than mastering the procedure.

24 + 15=

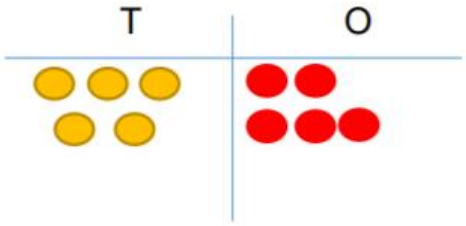
Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.

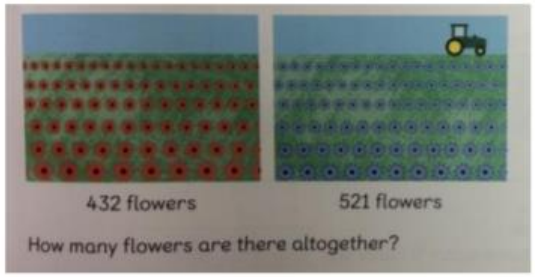


This representation prepares children for using column addition with formal recordings.



After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.





This formal recording should only be used as a guide for the procedure of written methods and not a way of calculating this sentence!

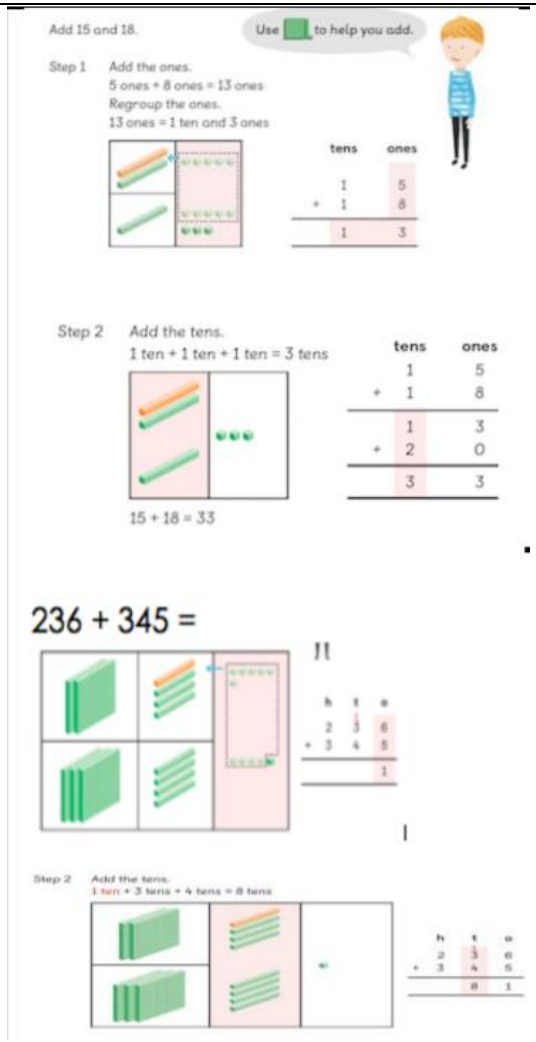
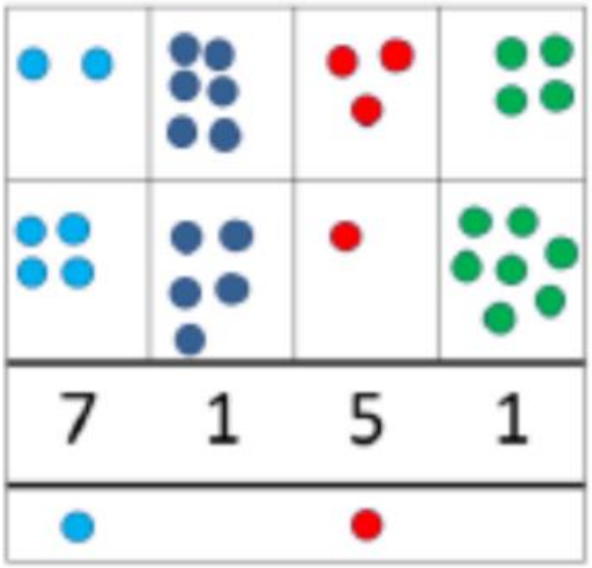
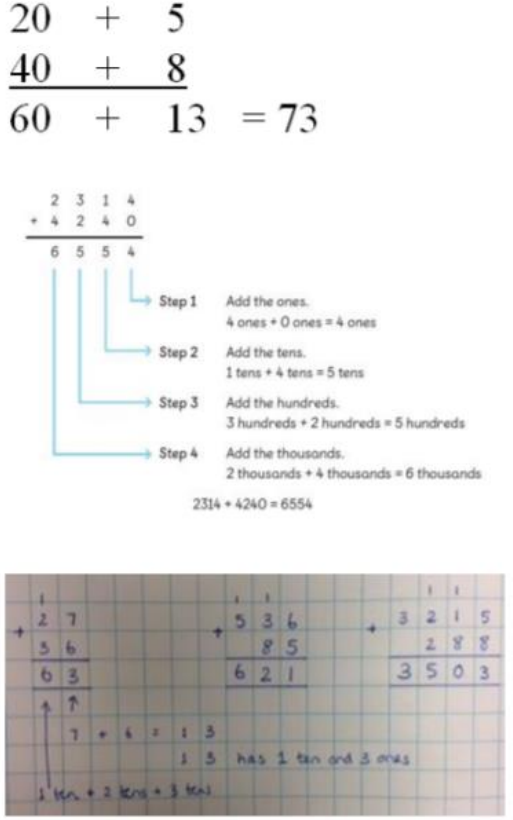
Calculations

21 + 42 =

$$\begin{array}{r} 21 \\ +\ 42 \end{array}$$

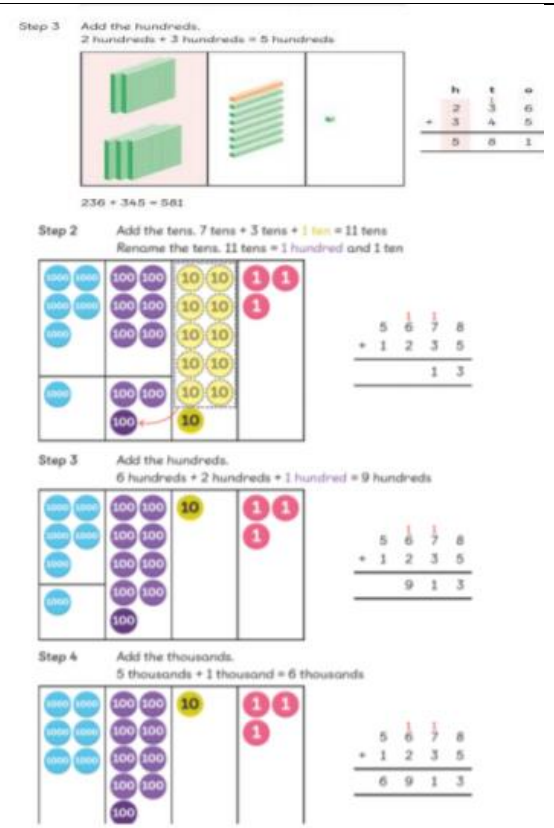
It is important that pupils are given plenty of (scaffolded) practice at choosing their own strategies to complete calculations efficiently and accurately. Explicit links need to be made between familiar number facts and the calculations that they can be useful for and pupils need to be encouraged to aim for efficiency.

345 + 30	274 - 50
1128 + 300	1312 – 300
326 + 342	856 - 724
945 + 1000	3892 – 1000
1482 + 900	2382 - 500

Column method regrouping Dienes or place value counters should be used as shown. Dienes blocks and place value grids should be used as shown in the diagrams. Even when working pictorially, pupils should have access to Dienes blocks.			

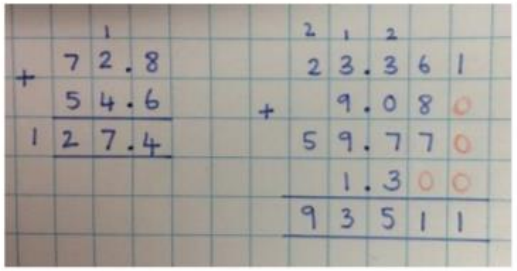
Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.

Start by partitioning the numbers before moving on to clearly show the exchange below the addition.



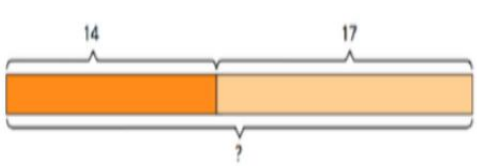
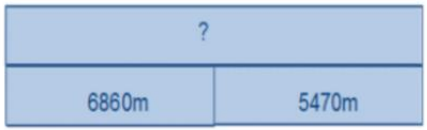
As children move on to decimals, money and decimal place value counters can be used to support learning.

As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

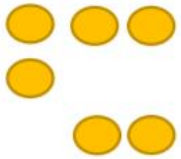
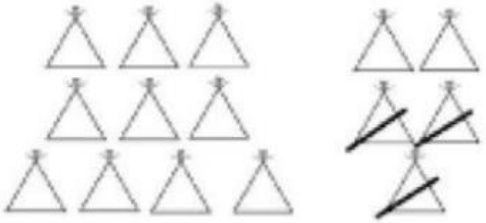
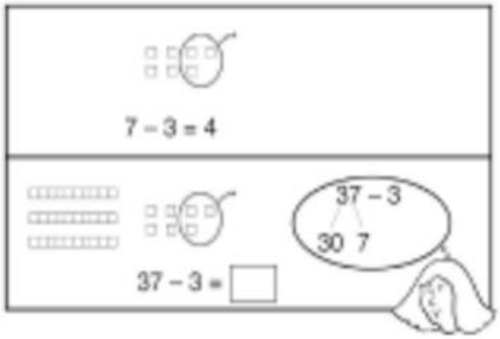


Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.

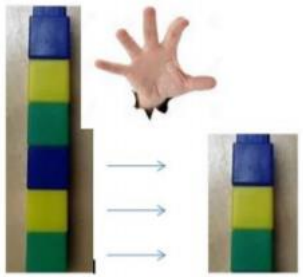
Empty decimal places should be filled with zero to show the place holder.

Using the bar to find missing digits. It is important for children to use the bar in this way to encourage the use of it to aid with problem solving. This is not a form of getting the correct answer but helping to guide children to the correct operation.	Helen has 14 bread-sticks. Her friend has 17. How many do they have altogether?   **Bar Model to support understanding of problem solving:**  A man sold 230 balloons at a carnival in the morning. He sold another 86 balloons in the evening . How many balloons did he sell in all?  Alison jogs 6,860 metres and Calvin jogs 5,470 metres. How far do they jog altogether? 

Subtraction

Objective and Strategies	concrete	Pictorial	Abstract
Taking away ones When this is first introduced, the concrete representation should be based upon the diagram. Real objects should be placed on top of the images as one-to-one correspondence so that pupils can take them away, progressing to representing the group of ten with a tens rod and ones with ones cubes.	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4 4 – 2 = 2  Taking away after counting out practical equipment. . Children would be encouraged to physically remove these using touch counting.  By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they don't have to keep recounting. They will then touch count the amount that are left to find the answer.	Cross out drawn objects to show what has been taken away.  15 – 3 = 12  28 - 4 =  7 – 3 = 4 37 – 3 30 7 37 – 3 =	18 - 3= 15 8 – 2 = 6

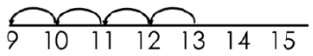
<table>
<tr>
<td></td>
<td>Those who are ready may record their own calculations

6 – 3 = 3
  </td>
<td></td>
<td></td>
</tr>
<tr>
<td>Counting back
Pupils should be encouraged to rely on number bonds knowledge as time goes on, rather than using counting back as their main strategy</td>
<td>Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.

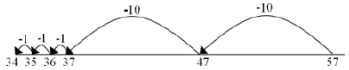
13 – 4
 

Use counters and move them away from the group as you take them away counting backwards as you go.

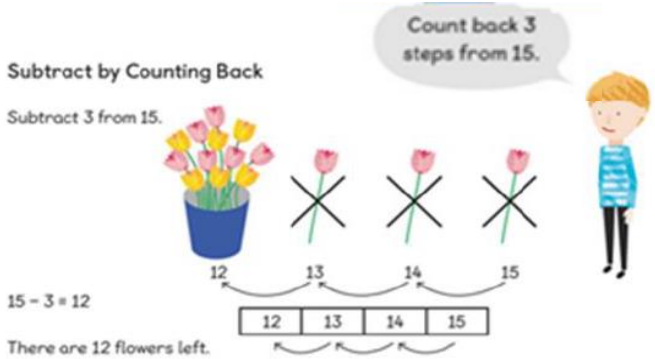
  </td>
<td>Count back on a number line or number track

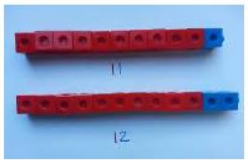
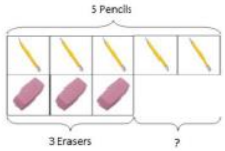
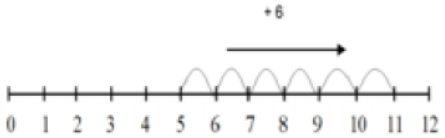
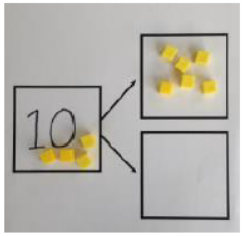
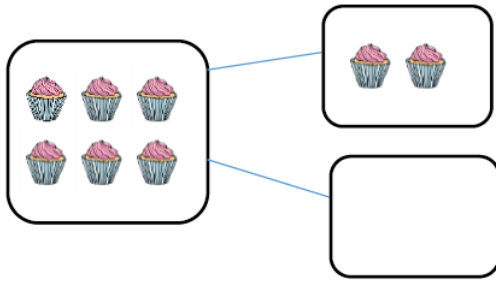
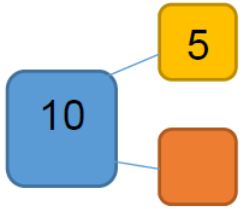
 

Start at the bigger number and count back the smaller number showing the jumps on the number line.

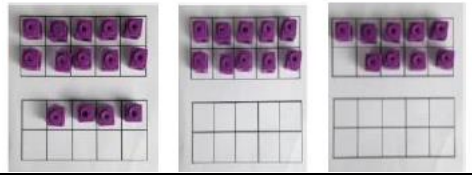
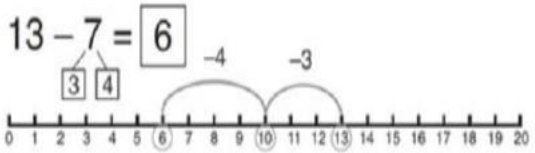
 

This can progress all the way to counting back using two 2 digit numbers.</td>
<td>Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

16 – 2 = 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td colspan="2"> 
</td>
</tr>
</table>

<table>
<tr>
<td>Find the difference</td>
<td>Compare amounts and objects to find the difference.
 
Use cubes to build towers or make bars to find the difference
 
Use basic bar models with items to find the difference</td>
<td>Count on to find the difference.
 
Draw bars to find the difference between 2 numbers.</td>
<td>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the numbers of sandwiches.</td>
</tr>
<tr>
<td>Part Part Whole Model
Teach both addition and subtraction alongside each other, as the pupils will use this model to identify the link between them. Pupils start with ten cubes placed on the whole.</td>
<td> 
Link to addition- use the part whole model to help explain the inverse between addition and subtraction.
If 10 is the whole and 6 is one of the parts. What is the other part?
10 - 6 =</td>
<td>Use a pictorial representation of objects to show the part part whole model.
 
 
7 – 5 = 2
2 boats are not red.</td>
<td> 
Move to using numbers within the part whole model.</td>
</tr>
</table>

<table>
<tr>
<td>Make 10
Single digit number from a 2-digit number Pupils identify how many need to be taken away to make ten first. Then they take away the rest to reach the answer.</td>
<td>14 – 9 =

Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.
  </td>
<td> 

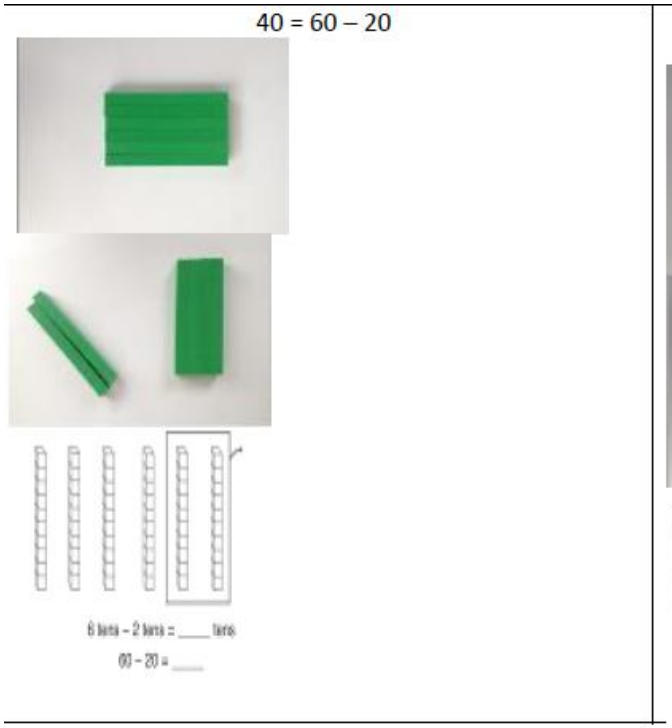
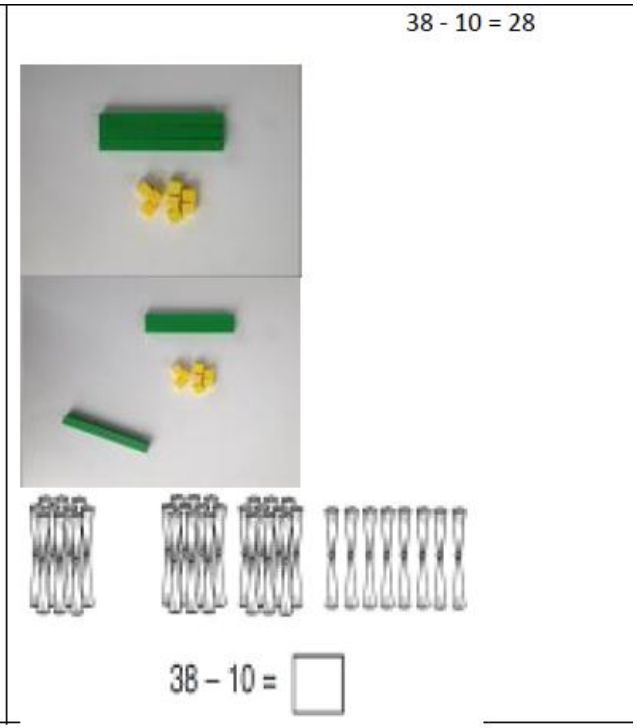
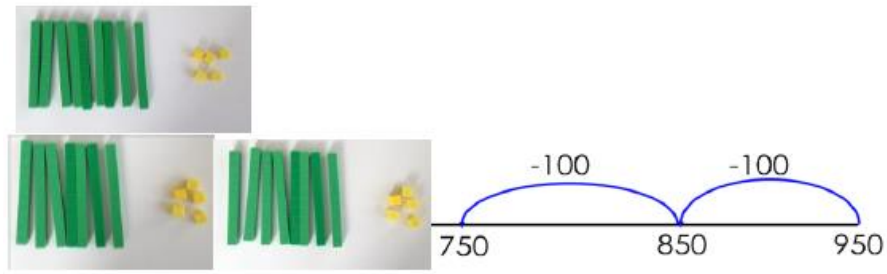
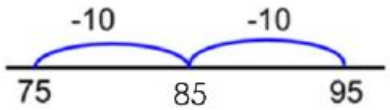
Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</td>
<td>16 – 8=

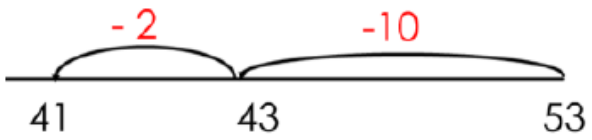
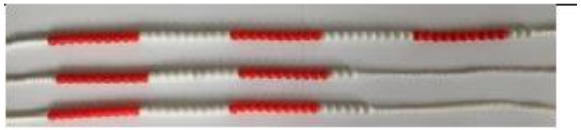
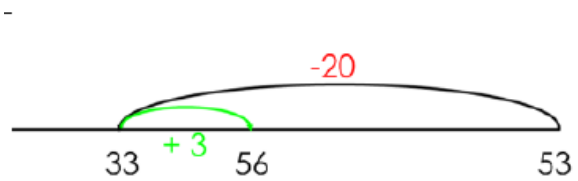
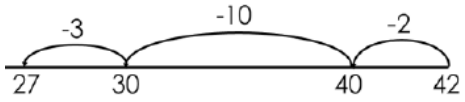
How many do we take off to reach the next 10?

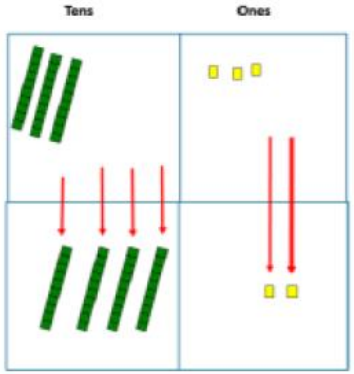
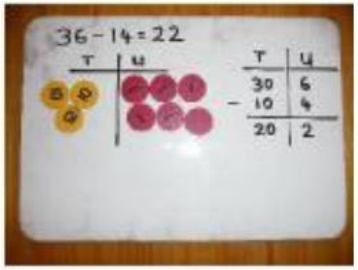
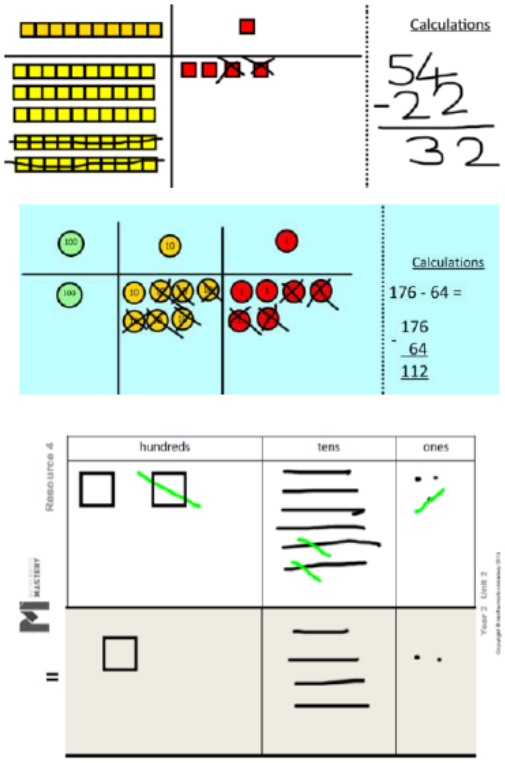
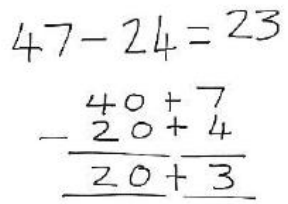
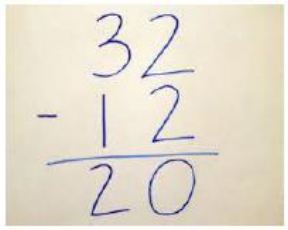
How many do we have left to take off?</td>
</tr>
<tr>
<td>Regroup a ten into 10 ones After the initial introduction, the Dienes blocks should be placed on a place value chart to support place value understanding. This will support pupils when they later use the column method.</td>
<td>  </td>
<td rowspan="2"> 
 

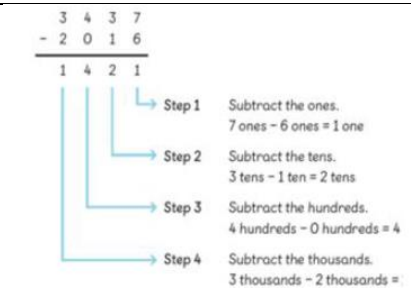
15 – 6 = 9

 
</td>
</tr>
<tr>
<td>Taking away from the tens Pupils should identify that they can also take away from the tens and get the same answer. This reinforces their knowledge of number bonds to 10 and develops their application</td>
<td>  </td>
</tr>
</table>

of number bonds for mental strategies.		
Subtracting multiples of ten - Counting back in multiples of ten and one hundred Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important as pupils need to understand that it is a ten not a one that is being taken away.	   	

Subtracting tens and ones Pupils must be taught to partition the second number for this strategy. Pupils will begin to see when this method is more efficient than subtracting tens and adding the extra ones, as shown.			53 – 12 = 41
Subtracting tens and adding extra ones Pupils must be taught to round the number that is being subtracted. Pupils will develop a sense of efficiency with this method, beginning to identify when this method is more efficient than subtracting tens and then ones.			53 - 17 = 36
Bridging through ten How pupils choose to apply this strategy is up to them. The focus should always be on efficiency.			42 - 15 = 2 13 10 3

Using known number facts Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy.			8 – 4 = 4 leads to 80 – 40 = 40 leads to 800 – 400 = 400
Column method without regrouping Pupils use the Dienes blocks (and later, images) to represent 3digit numbers but do not record a formal written method if there is no regrouping. Formal recording in columns is unnecessary for this mental strategy. It prepares them to subtract with 3-digits when regrouping is required.	Use Base 10 to make the bigger number then take the smaller number away.  Show how you partition numbers to subtract. Again make the larger number first.  	Draw the Base 10 or place value counters alongside the written calculation to help to show working.  	When children are not regrouping, partitioning is a mental strategy and does not need formal recording in columns.  This will help set out a clear written column subtraction but children should be encouraged to see the numbers and realise that a mental startegy is more efficient.. 



Step 1 Subtract the ones.

8 ones − 3 ones = 5 ones

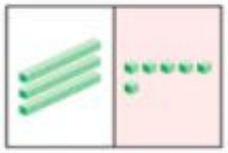
	tens	ones
	2	8
−		3
		5

Step 2 Subtract the tens.

	tens	ones
	2	8
−		3
	2	5

28 − 3 = 25

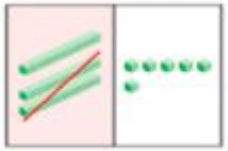
Step 1 Subtract the ones.



	tens	ones
	3	6
−	2	0
		6

Step 2 Subtract the tens.

3 tens − 2 tens = 1 ten



	tens	ones
	3	6
−	2	0
	1	6

36 − 20 = 16

The same would apply for three digit and four digits but you would add a hundred and thousands column.

Column method with regrouping

Strategy shows place value counters will need to be shown using dienes.

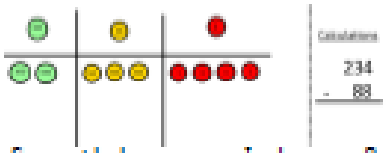
This example shows how pupils should work practically when being introduced to this method. There is no formal recording in columns in Year 1 but this practical work will prepare pupils for formal methods in Year 2.

Pupils are introduced to calculations that require two instances of regrouping (initially from tens to one and then from hundreds to tens).

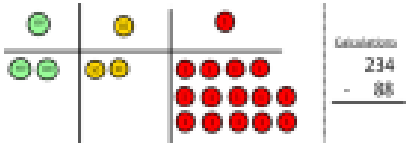
E.g. 232 – 157 and are given plenty of practice using concrete manipulatives and images alongside their formal written methods, ensuring that important steps are not

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

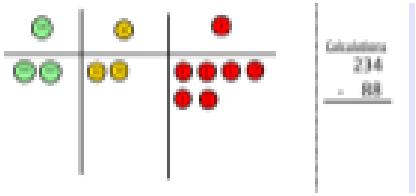
Make the larger number with the place value counters



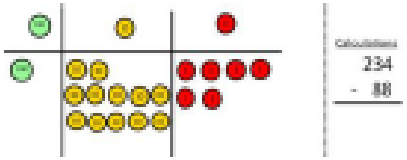
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



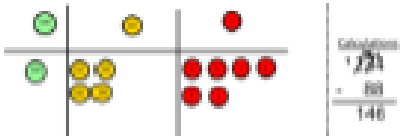
Now I can subtract my ones.



Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.

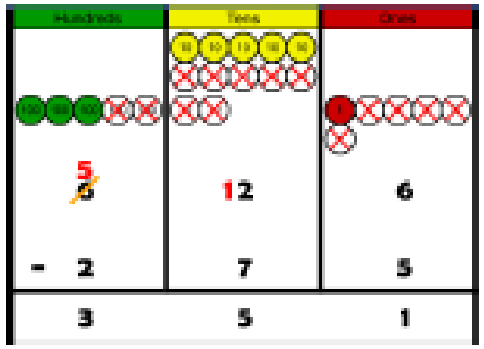


Now I can take away eight tens and complete my subtraction.



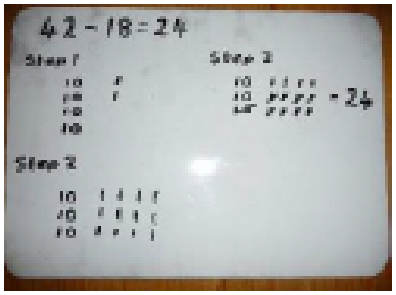
Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.



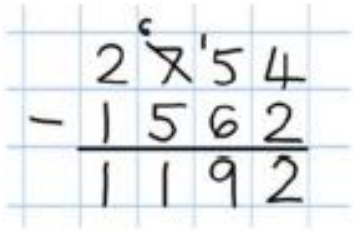
When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

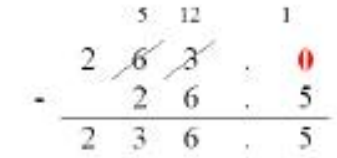


Children can start their formal written method by partitioning the number into clear place value columns.

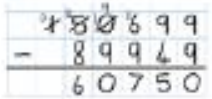
Moving forward the children use a more compact method.

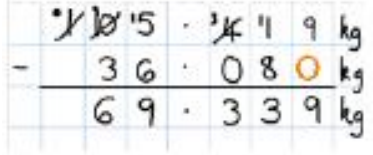


This will lead to an understanding of subtracting any number including decimals.

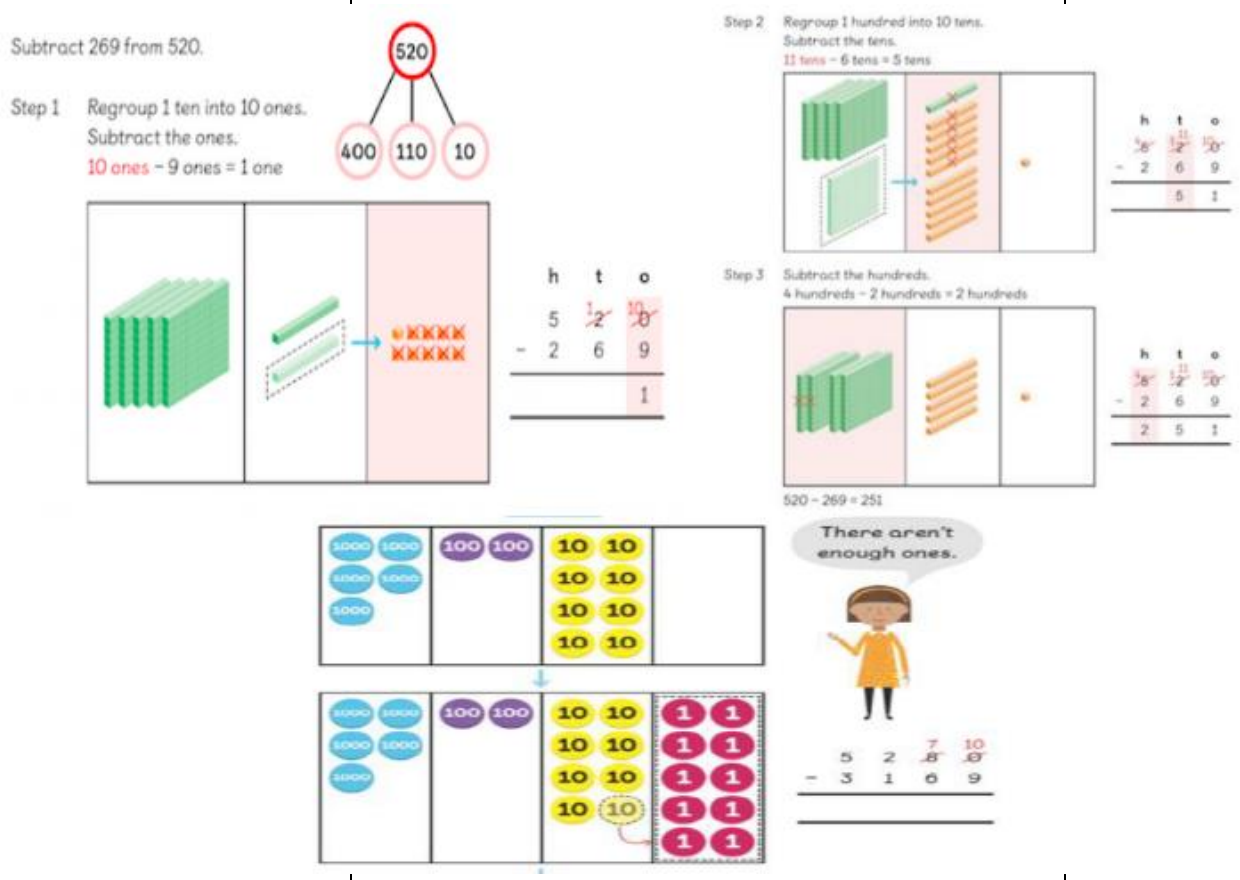


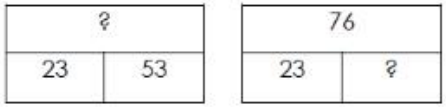
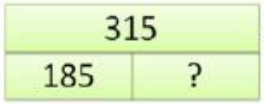
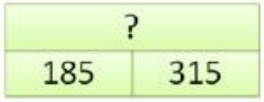
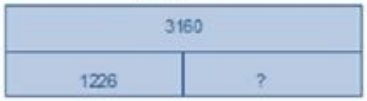
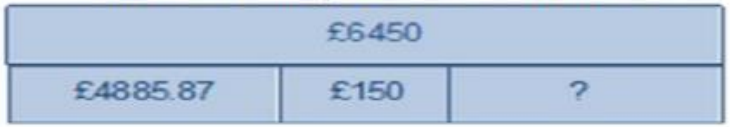
Very important to use in a range of contexts- measures and money.





missed in the recording. Caution should be exercised on introducing calculations requiring 'regrouping to regroup' (e.g. 204 – 137) ensuring ample teacher modelling using concrete manipulatives and images.



Recognise and use the inverse relationship between addition and subtraction	 Use this to check calculations and solve missing number problems.  315 – 185 = ? 185 + ? = 315  185 + 315 = ? ? – 185 = 315 There are 3,160 books in a shop. 1,226 are in English and the rest are in French. How many French books are there?  Chloe wants to buy a new car for £6450. She has £4885.87 in her savings account. Her Dad gives her £150 for her birthday. How much more money does she need to save? 		

Multiplication and Division

National Curriculum Objectives for Multiplication and Division

Year 1

- ✓ solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Year 2

- ✓ recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- ✓ calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
- ✓ show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- ✓ solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Year 3

- ✓ recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- ✓ write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two- digit numbers times one-digit numbers, using mental and progressing to formal written methods
- ✓ solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Year 4

- ✓ recall multiplication and division facts for multiplication tables up to 12 ×12
- ✓ use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- ✓ recognise and use factor pairs and commutativity in mental calculations
- ✓ multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- ✓ solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Year 5

Identify multiples and factors:

- ✓ identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers
- ✓ know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- ✓ establish whether a number up to 100 is prime and recall prime numbers up to 19
- ✓ multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- ✓ multiply and divide numbers mentally, drawing upon known facts divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- ✓ multiply and divide whole numbers and those involving decimals by 10,100 and 1000
- ✓ recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})
- ✓ solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes
- ✓ solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- ✓ solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates

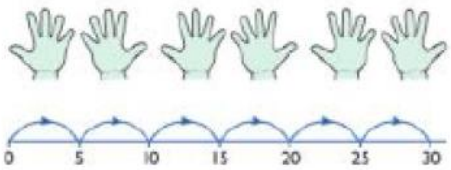
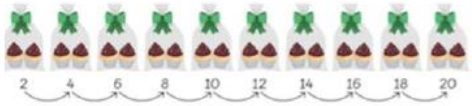
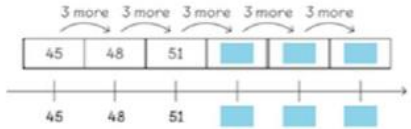
Year 6

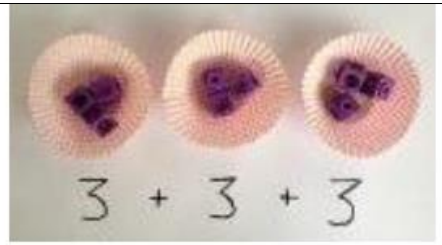
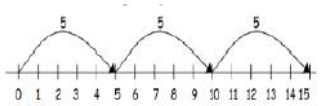
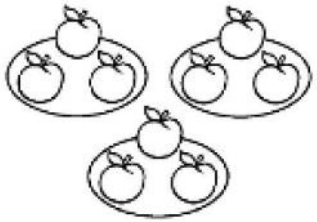
- ✓ multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- ✓ divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

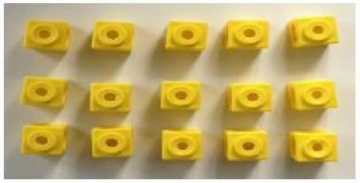
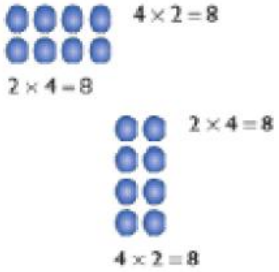
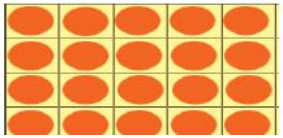
- ✓ divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- ✓ perform mental calculations, including with mixed operations and large numbers
- ✓ identify common factors, common multiples and prime numbers
- ✓ use their knowledge of the order of operations to carry out calculations involving the four operations
- ✓ solve problems involving multiplication and division
- ✓ use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

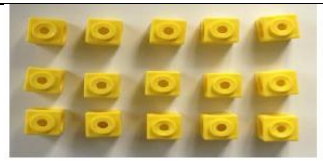
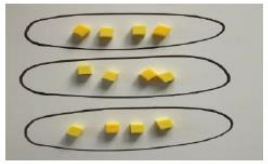
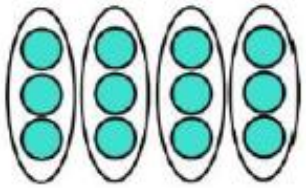
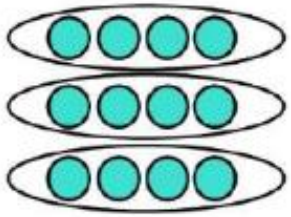
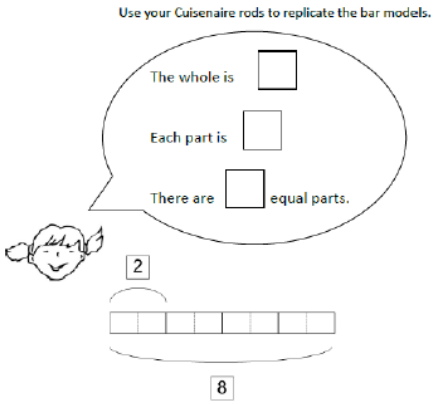
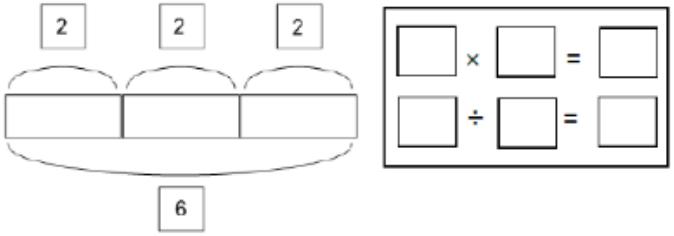
Progression in Calculations

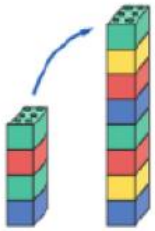
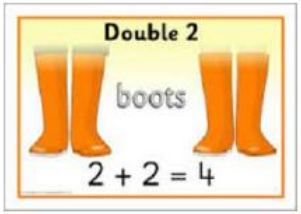
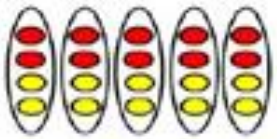
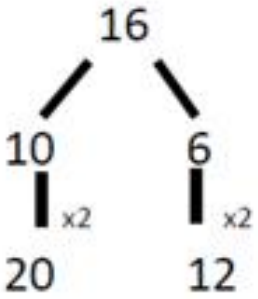
Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Counting in multiples The representation for the amount of groups supports pupils' understanding of the written equation. So two groups of 2 are 2, 4. Or five groups of 2 are 2, 4, 6, 8, 10. Count the groups as pupils are skip	  Count in multiples supported by concrete objects in equal groups.	Pupils can use their fingers as they are skip counting.  Use a number line or pictures to continue support in counting in multiples.  	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30 
Making equal groups and counting the total How this would be represented as an equation will vary. This could be 2 × 4 or 4 × 2. The importance should be placed on the vocabulary use alongside the equation. So this			

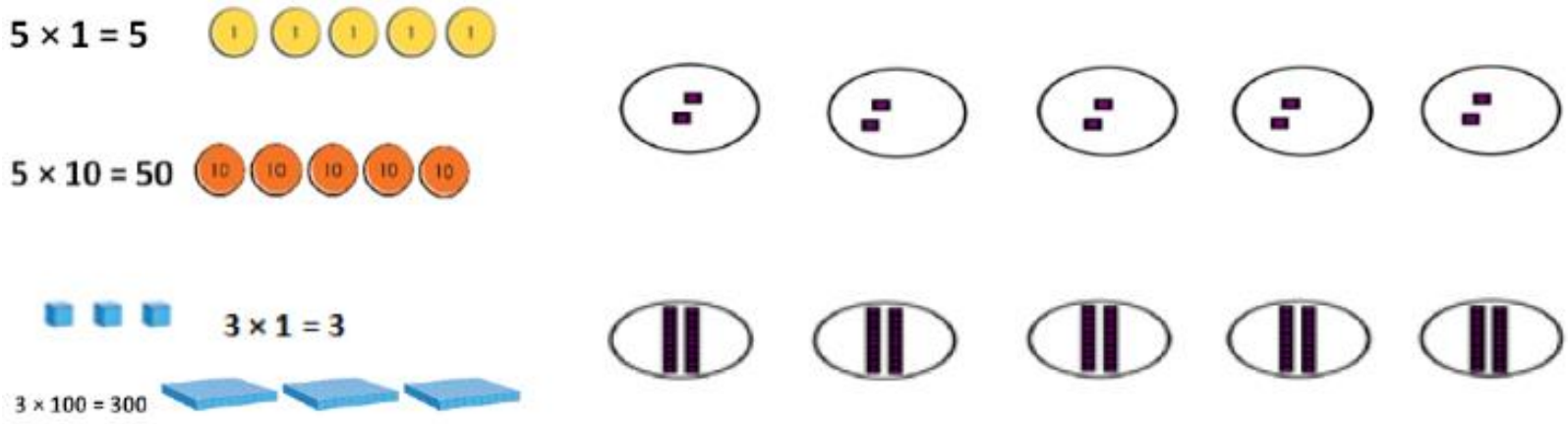
picture could represent 2 groups of 4 or 4 twice.	  Use different objects to add equal groups. 	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15  How many apples are there altogether?	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10

Arrays- showing commutative multiplication The relationship between multiplication and division also begins to be demonstrated. The multiple is the first number in a multiplication statement and the number of that multiple required is the second, so for example, five lots of four would be written 4 x 5.	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.  $4 \times 2 = 8$ $2 \times 4 = 8$ How many dots are there?  $2 \times 5 = 10$ $5 \times 2 = 10$ 2×5 is equal to 5×2. Link arrays to area of rectangles. 	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15 Pupils begin to understand multiplication in a more abstract fashion, applying their skip counting skills to identify the multiples of the 2x, 5x and 10x tables.

Multiplication is commutative Pupils should understand that an array and, later, bar models can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.	  	3 x 5 = ☐ 5 x 3 = ☐   3 x 4 = 12  4 x 3 = 12
Use of part-part whole model to establish the inverse relationship between multiplication and division This link should be made explicit from early on, using the language of the part-part-whole model, so that pupils develop an early understanding of the relationship between multiplication and	 Use your Cuisenaire rods to replicate the bar models. The whole is ☐ Each part is ☐ There are ☐ equal parts. 2 8	 What multiplication and division equations can you write for each bar model? Prove that the equations are correct using a bead string. 2 2 2 6 ☐ × ☐ = ☐ ☐ ÷ ☐ = ☐

division. Bar models with Cuisenaire rods) should be used to identify the whole, the size of the parts and the number of parts.			
Doubling to derive new multiplication facts Pupils learn that known facts from easier times tables can be used to derive facts from related times tables using doubling as a strategy. At this stage they double the 2x table facts to derive the 4x table facts	Use practical activities to show how to double a number.  double 4 is 8 4 × 2 = 8 	Draw pictures to show how to double a number. Double 4 is 8  5 × 4 = 20  5 × 2 = 10  Children will experience equal groups of objects. They will work on practical problem solving activities involving 	 Partition a number and then double each part before recombining it back together.

Ten times bigger
Multiplying by 10, 100 and 1000
Pupils' work on this must be firmly based on concrete representations – the language of ten times bigger must be well modelled and understood to prevent the numerical misconception of 'adding 0'.
Building on the ten times bigger work, pupils use appropriate Dienes blocks and place value counters to multiply 2, 3, 4, 5 and 10 by 10, 100 and 1000.



Building on the ten times bigger work, pupils use appropriate Dienes blocks and place value counters to multiply 2, 3, 4, 5 and 10 by 10, 100 and 1000.

Using known facts for multiplying by multiples of 10, 100 and 1000

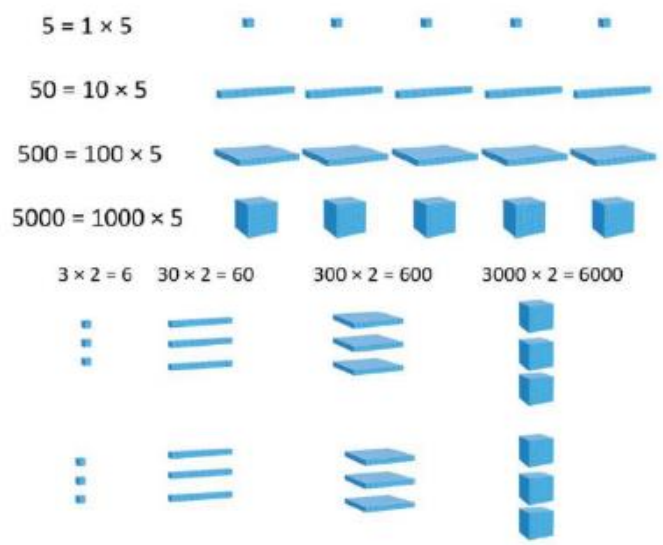
Pupils' growing understanding of place value, allows them to make use of known facts to derive multiplications using powers of 10.

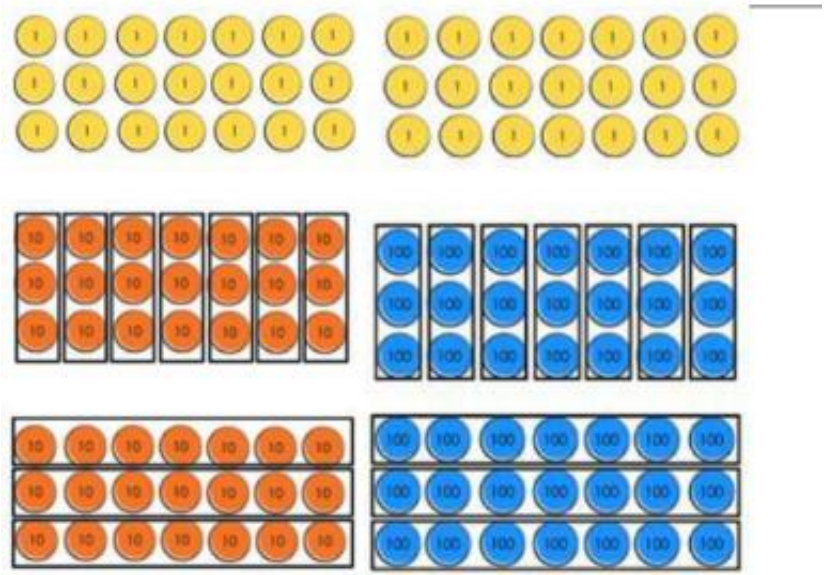
It is important to use tables with which they are already familiar (i.e. not 7 or tables in Year 3)

Emphasis is place on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far larger 'fact families' to be derived from a single known number fact.

Knowledge of commutativity is further extended and applied to find a range of related facts.

Grid method





7 × 3 = 21

7 × 30 = 210

70 × 3 = 210

70 × 30 = 2100

700 × 3 = 2100

7 × 300 = 2100

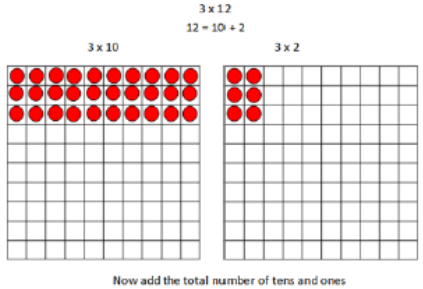
Grid method partitioning

Children should always consider whether partitioning is the best strategy – if it is possible to use strategies such as doubling (some may use doubling twice for ×4), they need to choose the most efficient strategy.

Children may wish to make jottings, including a full grid as exemplified here – but grid method is not a formal method and its only purpose is to record mental calculations. This supports the development of the necessary mental calculating skills but does not hinder the introduction of formal written methods in Year 4. Concrete manipulatives are essential to develop understanding. Using concrete manipulatives and later moving to using images that represent them, supports pupils' early understanding, leading towards formal written

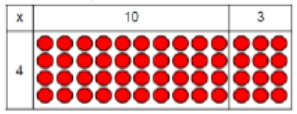
Show the link with arrays to first introduce the grid method.

Without regrouping-



×	10	2
3	=	∴

With regrouping-

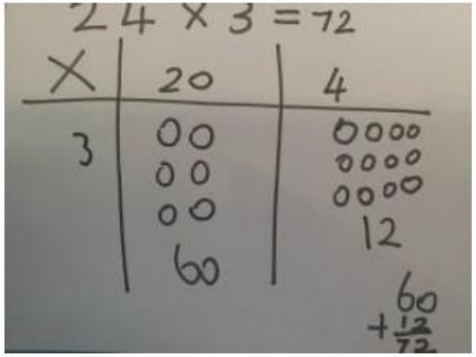


4 rows of 10
4 rows of 3

Move on to using Base 10 to move towards a more compact method.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Without regrouping -

×	10	2
3	30	6

3 x 12 = 36

Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

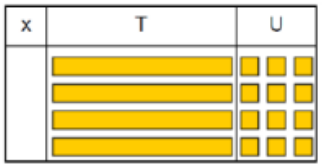
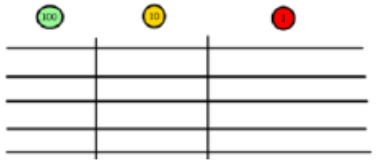
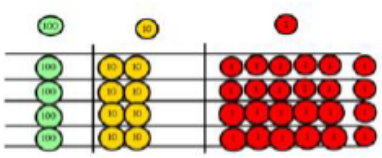
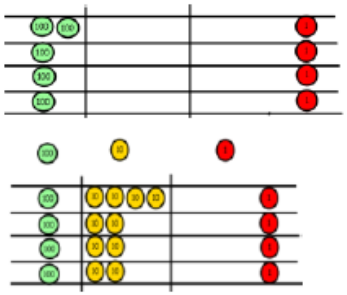
×	30	5
7	210	35

210 + 35 = 245

With regrouping - Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

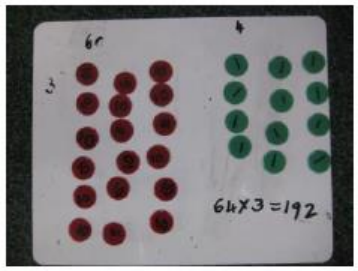
X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

methods in Year 4. Once again, this is a mental strategy, which they may choose to support with informal jottings, including a full grid, as exemplified here. Show the link with arrays to first introduce the grid method. Without regrouping- With regrouping- 4 rows of 10 4 rows of 3 Move on to using Base 10 to move towards a more compact method. 4 rows of 13 Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows. Fill each row with 126.	 4 rows of 13 Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.  Calculations 4 x 126 Fill each row with 126.  Calculations 4 x 126 Add up each column, starting with the ones making any exchanges needed. 		

Column multiplication

To begin with pupils are presented with calculations that require no regrouping or only regrouping from the ones to the tens. Their conceptual understanding is supported by the concomitant use of place value counters, both during teacher demonstrations and during their own practice. With practice pupils will be able to regroup in any column, including from the hundreds to the thousands, including being able to multiply numbers containing zero and regrouping through multiple columns in a single calculation. Children need to be taught to approximate first, e.g. for 72 x 38, they will use rounding: 72 x 38 is approximately 70 x 40 = 2800, and use the approximation to check the reasonableness of their answer.

Children can continue to be supported by place value counters at the stage of multiplication.



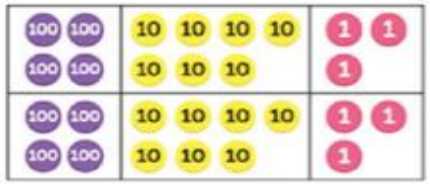
It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

228 × 3 =

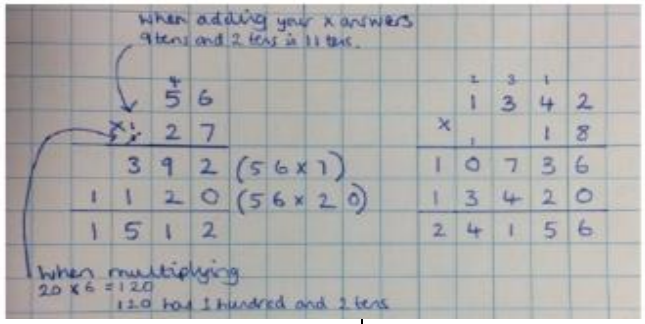
200 × 3 =
20 × 3 =
8 × 3 =

228
× 3

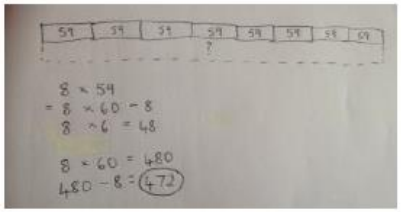
Short multiplication

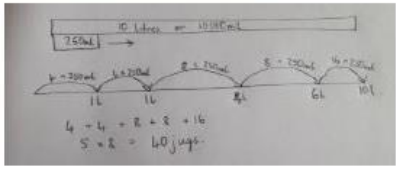


4 7 3
× 2



Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



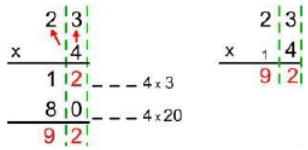


Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer. This moves to the more compact method.

32
x 24
8 (4 x 2)
120 (4 x 30)
40 (20 x 2)
600 (20 x 30)
768

23 x 4 = ?



Long multiplication –

Explain that first we are multiplying the top number by 7 starting with the units. (any carrying needs to be done underneath the numbers).

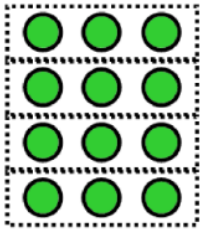
Now explain that we need to put a 0 underneath—explain that this is because we are multiplying the number by 20.. (2 tens) which is the same as multiplying 10 and 2.

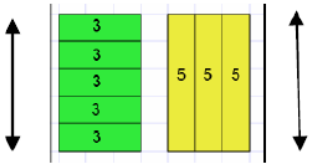
Now add the 2 numbers together to give you the answer.

This will need lots of modelling to show the children.

Bar modelling to represent the parts, the whole and the number of parts in multiplication word problems Cuisenaire rods can be used to create bar models that represent multiplications.

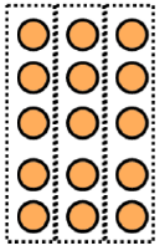
There are 4 bags of sweets with 3 sweets in each bag.
How many sweets are there altogether?





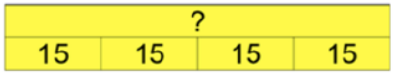
There are 3 school bags with 5 books in each one.

How many books are there altogether?



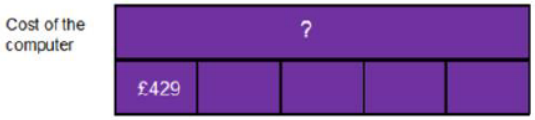
4 children go to the cinema. They each pay £15. How much do they spend altogether?

Whole unknown

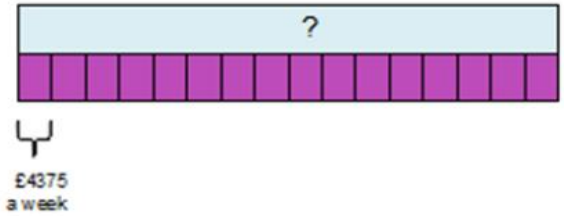


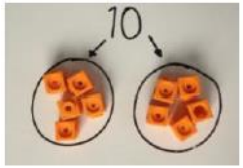
A computer costs 5 times as much as a television. The television costs £429.

How much does the computer cost?

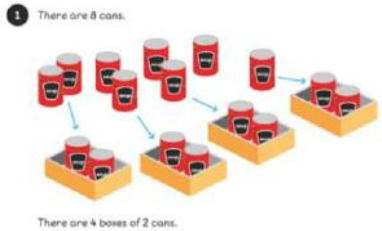
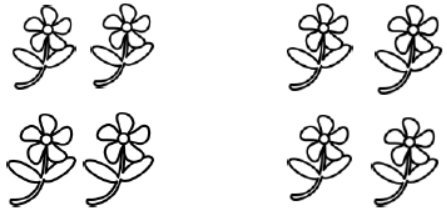
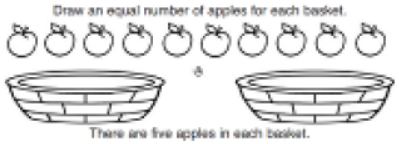


The cost to run a sports centre is £4375 a week, how much would it cost to run for 16 weeks?

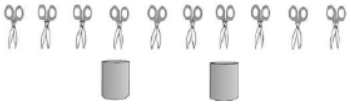


<table>
<tr><td>Division</td><td></td><td></td><td></td></tr>
<tr><td>Sharing objects into groups
Pupils should become familiar with division equations through working practically.</td><td>  
 

I have 10 cubes, can you share them equally in 2 groups?
Pupils should become familiar with division equation through working practically.
 

8 ÷ 4 = 2
 
</td><td>Children use pictures or shapes to share quantities.
 
8 ÷ 2 = 4
 

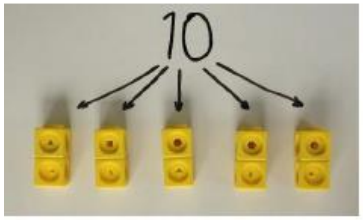
 

Here, division is shown as sharing. If we have ten pairs of scissors and we share them between two pots, there will be 5 pairs of scissors in each pot.
  </td><td>Share 9 buns between three people.

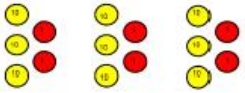
The division symbol is not formally taught in Year 1.

9 ÷ 3 = 3</td></tr>
<tr><td>Division as grouping</td><td></td><td></td><td></td></tr>
</table>

Divide quantities into equal groups.
Use cubes, counters, objects or place value counters to aid understanding.

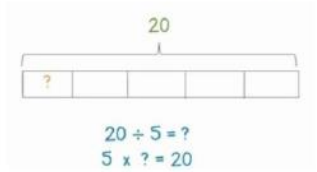


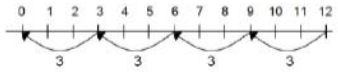
96 ÷ 3 = 32



Use a number line to show jumps in groups. The number of jumps equals the number of groups.

Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.





Here, division is shown as grouping. If we have ten forks and we put them into groups of two, there are 5 groups.

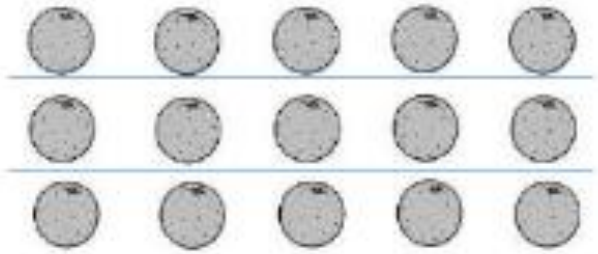
28 ÷ 7 = 4

Divide 28 into 7 groups. How many are in each group?

Use of part-part-whole model to represent division equations and to emphasise the relationship between division and multiplication Pupils use arrays of concrete manipulatives and images of familiar objects to find division equations. They begin to use dot arrays to develop a more abstract concept of division.

Link division to multiplication by creating an array and thinking about the number sentences that can be created.

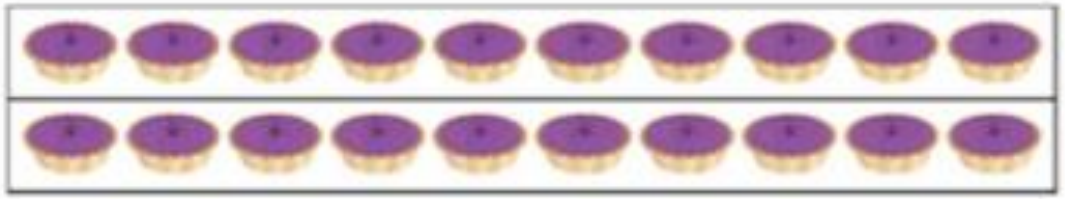
Eg 15 ÷ 3 = 5 5 x 3 = 15
15 ÷ 5 = 3 3 x 5 = 15



Draw an array and use lines to split the array into groups to make multiplication and division sentences.

20 ÷ 4 = ☐ 20 ÷ 5 = ☐

Make a family of multiplication and division facts.

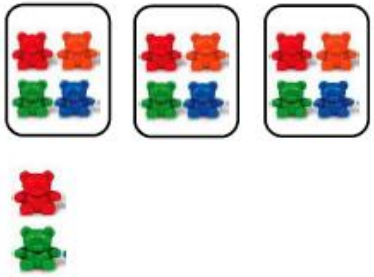
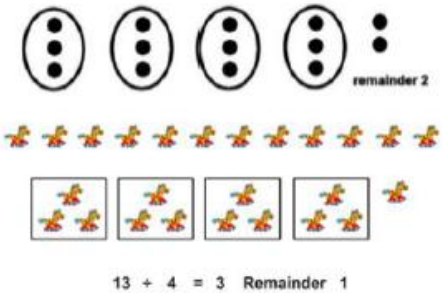


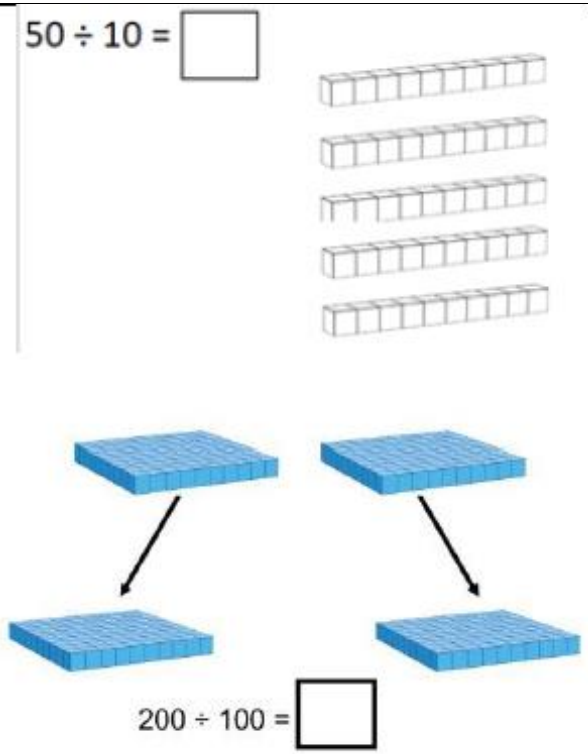
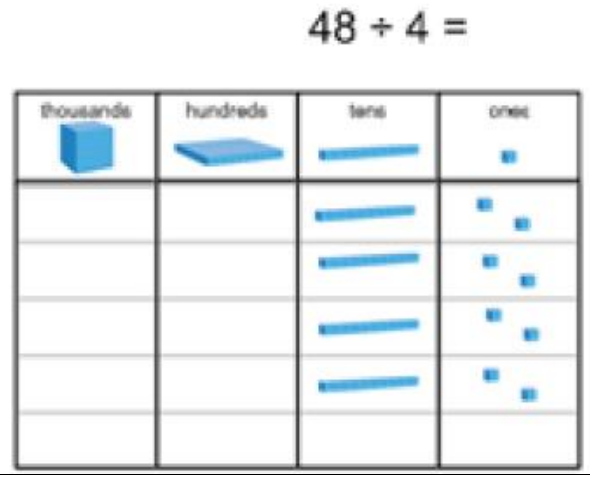
2 x 10 = 20 ——— 20 ÷ 10 = ☐

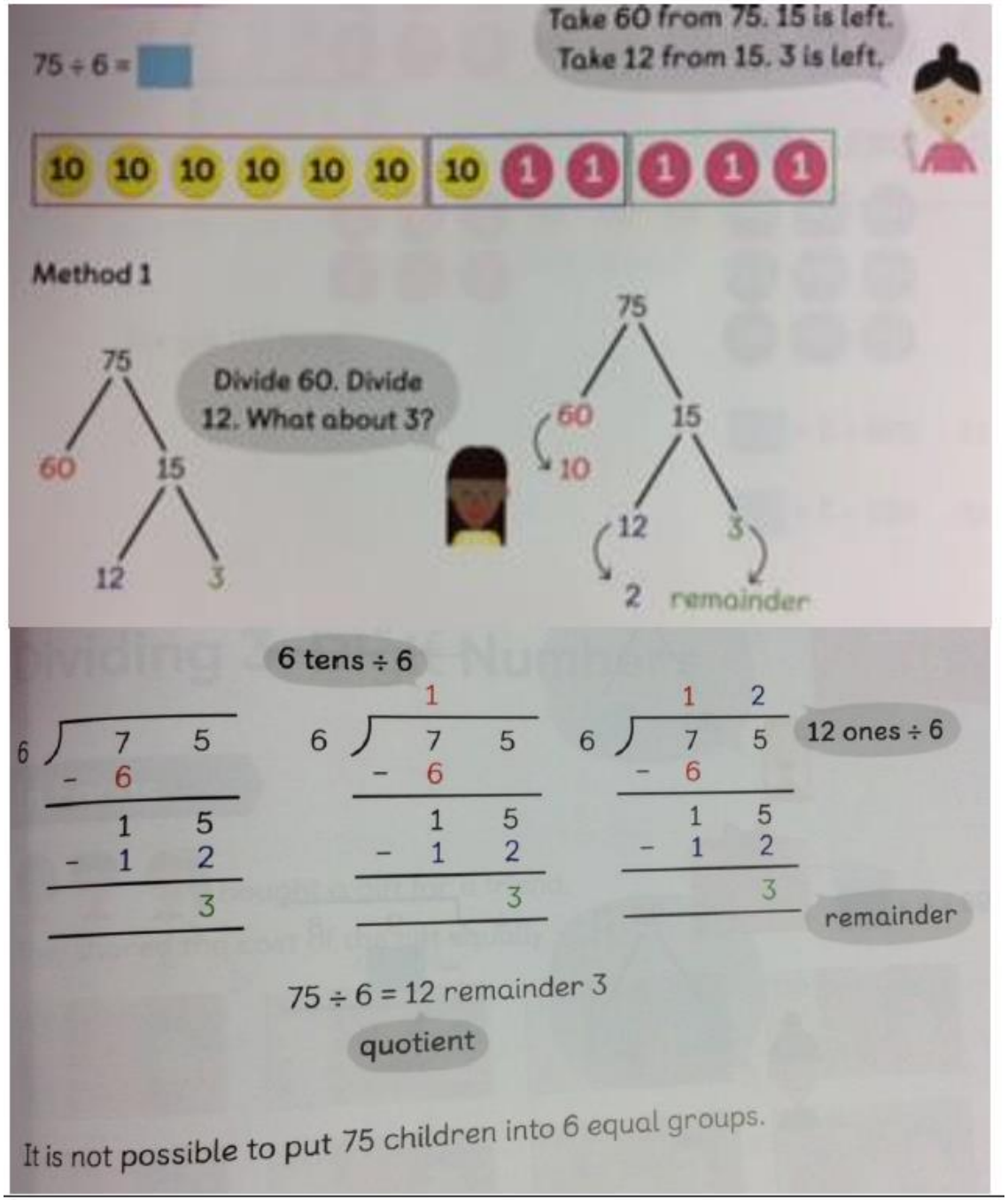
10 x 2 = 20 ——— 20 ÷ 2 = ☐

Find the inverse of multiplication and division sentences by creating four linking number sentences.

7 x 4 = 28
4 x 7 = 28
28 ÷ 7 = 4
28 ÷ 4 = 7

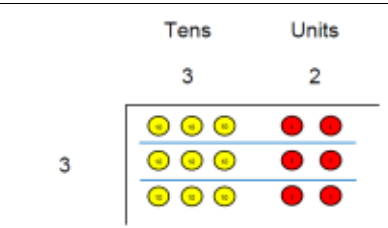
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  13 ÷ 4 = 3 Remainder 1	Complete written divisions and show the remainder using r. 29 ÷ 8 = 3 REMAINDER 5 dividend divisor quotient remainder

Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 Pupils use the strategy of sharing into equal groups of tens, hundreds or thousands to reinforce their understanding of place value and the concept of division as sharing into equal groups. They master this skill with calculations where no partitioning is required, to prepare them for the next step	
Division by partitioning and sharing into equal groups with no regrouping required Pupils apply the skills of sharing into equal groups, dividing 2-digit numbers by 3, 4 and 5. They should be encouraged not to use this strategy to divide by 2 (using halving instead).	

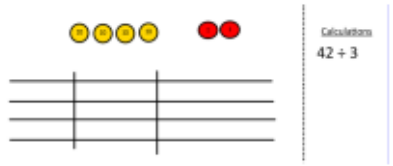

75 ÷ 6 =
Take 60 from 75. 15 is left.
Take 12 from 15. 3 is left.
10 10 10 10 10 10 10 1 1 1 1 1
Method 1
75
60 15
12 3
Divide 60. Divide 12. What about 3?
75
60 15
10
12 3
2 remainder
6 tens ÷ 6
6) 7 5
- 6
1 5
- 1 2
3
1
1 2
12 ones ÷ 6
remainder
75 ÷ 6 = 12 remainder 3
quotient
It is not possible to put 75 children into 6 equal groups.

Short division

In Year 4 Short division of 4-digit numbers by 1-digit numbers. Pupils start with dividing 4-digit numbers by 2, 3 and 4, where no regrouping is required. Place value counters are used simultaneously in a place value chart, to develop conceptual understanding. They progress to calculations that require regrouping in the hundreds or tens columns. Pupils build on their conceptual knowledge of division to become confident with dividing numbers where the tens digit is smaller than the divisor, extending this to any digit being smaller than the divisor.

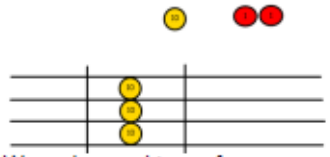


Use place value counters to divide using the bus stop method alongside

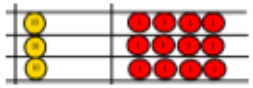


42 ÷ 3=

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

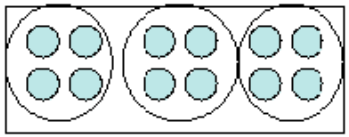


We exchange this ten for ten ones and then share the ones equally among the groups.

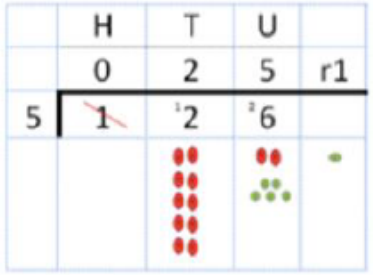


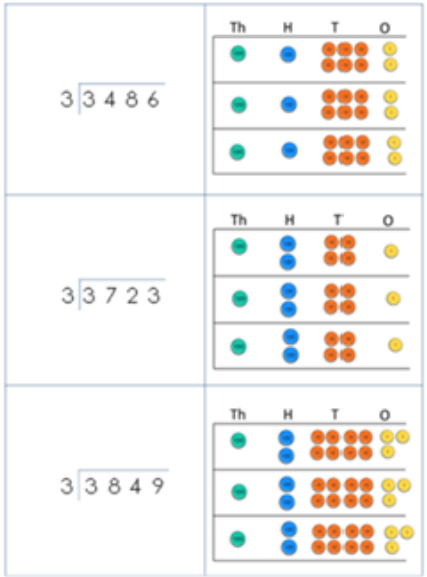
We look how much in 1 group so the answer is 14.

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.

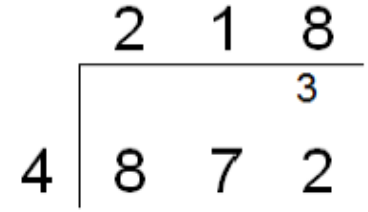


Encourage them to move towards counting in multiples to divide more efficiently.

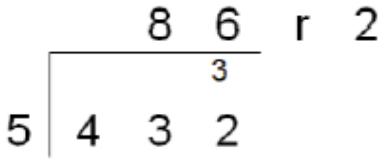


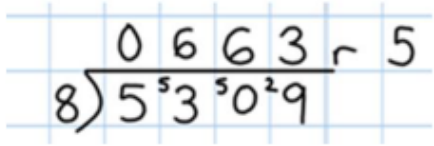


Begin with divisions that divide equally with no remainder.

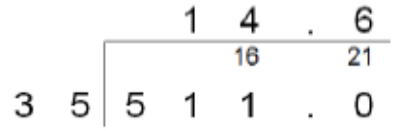


Move onto divisions with a remainder. Short division with remainders: Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where pupils consider the meaning of the remainder and how to express it, ie. as a fraction, a decimal, or as a rounded number or value , depending upon the context of the problem.



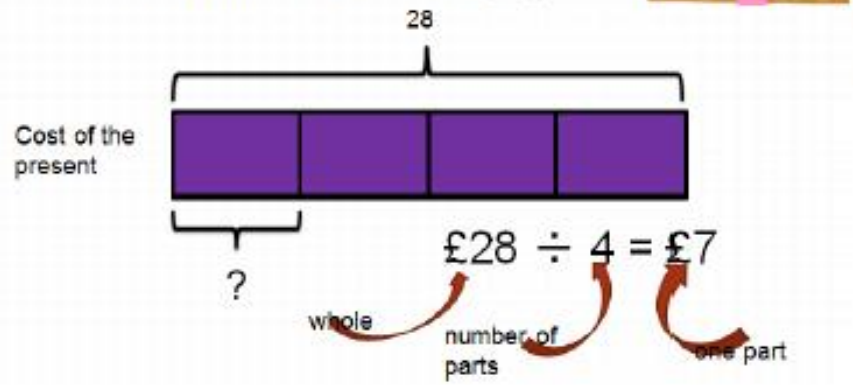


Finally move into decimal places to divide the total accurately.

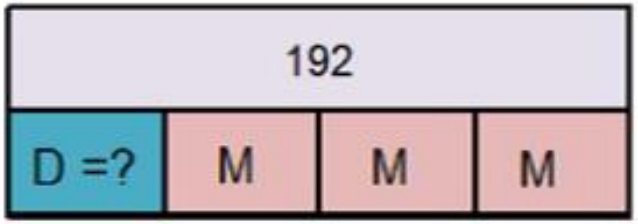


Using the bar to aid the solving of division problems.

Four children bought a present for £28. They shared the costs equally. How much did each child pay?



Desmond and Melissa collect cards. They have 192 cards in all. Melissa has three times as many cards as Desmond. How many cards does Desmond have?



Bar Model to support understanding of problem solving:

Frank has 4920 apples. He needs to put them into baskets of 40. How many baskets does he need?